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ABSTRACT

A recurring problem in educational research is how to account for non-random selection that has restricted the range of the variables of interest in correlational analyses. Several expressions due to H. Pearson (1903) and presented in matrix notation by D. N. Lawley (1943-44) are commonly used in selection settings to adjust for samples chosen on the basis of a direct selection variable. When validity evidence is provided for tests used to select applicants or for tests administered to examinees selected for some other reasons, correlations are often used. On the basis of this study, two suggestions can be made to practitioners using the Pearson-Lawley adjustments in these settings. The first is a recommendation for the use of the estimates for the standard error of the adjusted correlations. The second is a warning concerning the use of an adjusted correlation without reporting an estimate of its standard error. Since these suggestions are made in the context of an exact fit of the data to the Pearson-Lawley assumptions of linearity and homogeneity of variance, they indicate "best case" results for real data that may not fit the assumptions exactly. A computer simulation contributed new information about standard errors of adjusted correlations in three areas. Nine tables are included. (TJH)

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Standard Errors of Correlations Adjusted for Incidental Selection

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Introduction

A recurring problem in educational research is how to take into account non-random selection that has restricted the range of the variables of interest in correlational analyses. This problem has important implications for policy and practice in education and in testing (Linn, 1983). Several expressions due to Pearson (1903) and presented in matrix notation by Lawley (1943-44) are commonly used in selection settings to adjust for samples chosen on the basis of a direct selection variable. These expressions include one in which the correlation in a population before selection is estimated by adjusting a correlation from a sample selected directly on the basis of one of the variables of interest (two-variable case, direct or explicit selection). Another of the Pearson-Lawley formulas adjusts a correlation from a sample selected on the basis of a variable different from the two variables of interest to estimate a correlation in the population before selection (three-variable case, indirect or incidental selection). The formula for this adjustment is

$$R_{xy} = \frac{r_{xy} + (\{S_z/s_z\}^2-1)r_{xz}r_{yz}}{\sqrt{[1 + (\{S_z/s_z\}^2-1)r_{xz}^2][1 + (\{S_z/s_z\}^2-1)r_{yz}^2]}}$$

where lower-case r's and s's denote correlations and standard deviations for the selected sample while upper-case R's and S's denote correlations and standard deviations for the total population. Observations are selected if the value of the variable z is smaller or larger than a criterion value and the variables x and y are indirectly influenced by the selection.

The Pearson-Lawley relationships between correlations for a selected sample and correlations for the full population hold if two assumptions are met. The first assumption is that there is a linear relationship between the

selection variable and the other variables of interest. The second assumption is homoscedasticity, that the variances of the indirectly selected variables and the covariances of those variables with other variables do not depend on the value of the selection variable. For the two-variable case, this means that the slopes of the regressions for the selected sample and for the full population are equal ($b_{xz} = B_{xz}$) and that the standard deviations of the indirectly selected variable conditioned on the selection variable for the selected sample and for a random sample are equal ($s_{x.z} = S_{x.z}$). For the three-variable case, the slopes of the regressions of the two indirectly selected variables for the selected sample and for the full population are not necessarily equal ($b_{xy} \neq B_{xy}$).

Most of the previous research concerning these formulas has examined the bias of the adjusted correlations when the assumptions of the Pearson-Lawley formulas have not been met or when the selection process is poorly modeled by the formulas. Another area of research that is imperative for the appropriate use of these adjusted correlations, but which has received less attention is the variability of these estimates.

Despite the importance of standard errors (SE's) of correlations, relatively few estimates of the SE's of correlations adjusted with the Pearson-Lawley formulas have been derived or examined. For the two-variable case, a large-sample estimate of the SE of an adjusted correlation was first derived by Kelley (1923). A similar estimate was later derived by Bobko and Rieck (1980).

The principal differences between the early work of Kelley and the more recent work of Bobko and Rieck involve: (1) the method used to derive the SE estimates, (2) the form of the estimate, and (3) the estimate of the SE of the

unadjusted correlation used in the expression for the SE. An advantage of the delta method used by Bobko and Rieck to derive the SE estimate of an adjusted correlation is that asymptotic covariances of correlations required in more complicated cases involve only correlations with one variable in common. The estimate of the SE of the unadjusted correlation used by Bobko and Rieck is appropriate for bivariate normal or, at least, bivariate mesokurtic variables in the part of the population fitting the selection criterion. Kelley suggested the use of an "appropriate" estimate of the SE of an unadjusted correlation and provided an estimate dependent on the kurtoses of the distributions for the two variables of interest. This more general expression is appropriate if the relationship between the variables in the part of the population fitting the selection criterion is linear, as is true if the assumptions of the Pearson-Lawley adjustments are met.

Other studies of the SE's of correlations adjusted for range restriction include Gullickson and Hopkins (1976), Brandt, McLaughlin, Wise and Rossmeissl (1984), and Mendoza, Hart and Powell (1987). Gullickson and Hopkins (1976) calculated empirical nomograms for confidence intervals of adjusted correlations for the two-variable case. Brandt, et al. (1984) used a large data set containing ASVAB scores and training grades to develop bootstrap estimates of the SE's of adjusted correlations. They intended these to be useful descriptive statistics in situations where the Pearson-Lawley adjusted correlations were to be reported as part of a criterion-related validity study. Mendoza, Hart and Powel (1987) developed empirical confidence intervals for the correlation adjusted using the two-variable formula when the Pearson-Lawley assumptions were violated.

SE's of correlations adjusted for the more complex selection situations, including the three-variable case, have not been investigated thoroughly. Accordingly, the primary purpose of this study was to investigate SE estimates for correlation coefficients adjusted using the Pearson and Lawley methods. Specifically, this investigation had three major purposes: (1) to provide large-sample approximations of the SE of a correlation adjusted using the Pearson-Lawley three-variable correction formula; (2) to examine the SE's of adjusted correlations under specific conditions; and (3) to compare various estimates of the SE's under direct and indirect selection.

Method

Two theory-based large-sample estimates of the SE of a correlation adjusted for indirect selection were developed using the delta method (based on Taylor series expansions for variances and on formulas for the asymptotic variances and covariances of correlations). As assumed by Kelley and by Bobko and Rieck, S_z/s_z was considered fixed in this application of the delta method. The use of the delta method in this setting was appropriate since each unadjusted sample correlation had a variance of order N^{-1} and the formula for the three-variable adjustment was differentiable at the selected population correlations. Both SE estimates found using the delta method are of the form

$$SE(R_{xy}) = [(R'_{xy})^2 var(r_{xy}) + (R'_{xz})^2 var(r_{xz}) + (R'_{yz})^2 var(r_{yz}) + 2R'_{xy}R'_{xz}cov(r_{xy}, r_{xz}) + 2R'_{xy}R'_{yz}cov(r_{xy}, r_{yz}) + 2R'_{xz}R'_{yz}cov(r_{xz}, r_{yz})]^{1/2}$$

where R'_{xz} is the derivative of the adjusted correlation R_{xy} with respect to the unadjusted correlation r_{xz}, for example. These derivatives are shown in Table 1.

Table 1

Partial Derivatives of the Adjusted Correlation with respect to Each Unadjusted Correlation

	Unadjusted Correlation	Derivatives of R_{xy} Evaluated at Estimates of the Parameters
R'_{xy}	r_{xy}	$[1 + W(r_{xz})^2]^{-1/2}[1 + W(r_{yz})^2]^{-1/2}$
R'_{xz}	r_{xz}	$W[1 + W(r_{xz})^2]^{-3/2}[1 + W(r_{yz})^2]^{-1/2}[r_{yz} - r_{xy}r_{xz}]$
R'_{yz}	r_{yz}	$W[1 + W(r_{xz})^2]^{-1/2}[1 + W(r_{yz})^2]^{-3/2}[r_{xz} - r_{xy}r_{yz}]$

Note: $W = (S_z/s_z)^2 - 1$

The difference between the two estimates of the SE of the adjusted correlation is in the estimates chosen for the variances and covariances. Table 2 contains two sets of estimates for the necessary asymptotic variances and covariances of the unadjusted correlations. The variance and covariance estimates fitting the set A assumptions correspond to the estimate of the variance of the unadjusted correlation used by Bobko and Rieck. The assumptions of this first estimate of the SE of the adjusted correlation probably are not strictly met in real settings involving non-random selection. The variance and covariance estimates fitting the set B assumptions correspond to the estimate of the variance of the unadjusted correlation suggested by the work of Kelley. The more general assumptions of this second estimate of the SE of the adjusted

Table 2

Estimated Sampling Variances and Covariances
of Unadjusted Correlations

Statistic	Estimated Sampling Variance or Covariance	Set of Assumptions
$\mathrm{var}(r)$	$(1 - r^2)^2/N$	A
	$[(1 - r^2)^2/N][1 - \frac{1}{4}(K + K' - 6)r^2/(1 - r^2)]$*	B
$\mathrm{cov}(r_{xz}, r_{yz})$	$N^{-1}[r_{xy}(1 - r_{xz}^2 - r_{yz}^2) - \frac{1}{2}r_{xz}r_{yz}(1 - r_{xy}^2 - r_{xz}^2 - r_{yz}^2)]$	A
	expression in terms of 2nd and 4th moments and correlations**	B

* K = kurtosis for 1st variable

= (fourth moment)/(second moment)^2 for 1st variable

K′= kurtosis for 2nd variable

** This expression is given in Allen (1987).

Note: Assumptions

A 1. error/N to 2nd and 3rd powers is negligible

2. linearity of regression

3. mesokurtosis of the joint distribution of explicit and implicit variables (e.g. multivariate normality)

B 1. error/N to 2nd and 3rd powers is negligible

2. linearity of regression

Adapted from Isserlis (1916).

correlation are more likely to be met than are the assumptions of the first estimate. However, the kurtoses of the distributions of the variables of interest may be difficult to estimate, so there was some interest in assessing whether a practical difference existed.

These two large-sample estimates of the SE of an adjusted correlation in the three-variable case are complicated. In particular, if normality or at least mesokurtosis of the selected sample is not assumed, higher order moments are needed to estimate the SE of an adjusted correlation. However, neither of these estimates requires sophisticated optimization routines or is computer intensive. Modern computer capabilities make all of these calculations possible and convenient.

For the three-variable case, these two estimates were compared to one another, to a bootstrap estimate based on 100 bootstrap samples and to an empirical standard deviation of a series of 250 adjusted correlations generated in a simulation study. Because the data generation procedure made it convenient, estimates of the SE of adjusted correlations were also compared for the two-variable case. These two-variable estimates were Bobko and Reick's estimate, Kelley's estimate, a bootstrap estimate, again based on 100 bootstrap samples, and an empirical standard deviation. All estimates were examined in the metric of the correlation.

The simulation study manipulated factors defined by (1) distributional characteristics (multivariate normal, multivariate platykurtic), (2) population correlations, (3) selection ratio, and (4) size of the selected sample, in situations that satisfied the basic assumptions of linearity and homoscedasticity. The data for both the three-variable and two-variable

situations were taken from the same samples, generated to fit trivariate distributions. The samples were chosen from six populations. Three of these populations had multivariate normal distributions, while three had multivariate platykurtic distributions. A standardized beta (2,2) distribution was chosen as the basis for the platykurtic populations because the kurtosis for this distribution, 2.14, was reasonably close to the median kurtosis of 2.39 found for actual achievement test data by Brandenburg (1972/1973). The beta distribution is also curtailed at both ends of the distribution in a way not unlike distributions of test scores. Three population correlation matrices, seen in Table 3, were used to generate the data from the two types of distributions. For the three-variable case, the population correlation values that were estimated were .25, .50 and .75, while for the two-variable case, the estimated population values were .20, .50 and .80. Each of the data sets contained approximately 10, 50 or 90 percent of the original simulated observations and 50, 100 or 500 selected observations. The above set of conditions represented 54 combinations of the four design parameters included in the simulation study, and each set of conditions was replicated 250 times. As stated previously, each of the data sets fit the assumptions required for use of the Pearson-Lawley formulas.

Table 3

Unselected Population Correlation Matrices

$\underline{P}_1$				$\underline{P}_2$			$\underline{P}_3$		
	Z	X	Y	Z	X	Y	Z	X	Y
Z	1.00	.20	.30	1.00	.50	.80	1.00	.80	.95
X		1.00	.25		1.00	.50		1.00	.75
Y			1.00			1.00			1.00

Z = the explicit selection variable
X = an incidental selection variable (e.g. test score)
Y = an incidental selection variable (e.g. criterion variable)

Results

The computer simulation contributed new information about the SE's of adjusted correlations in three areas. Tables in the current section will present examples of the results in these three areas. Tables presenting the average estimates of the SE's of the adjusted correlation for the three-variable case and for the two variable case under all 54 experimental conditions are in the appendix.

First, a comparison of different estimates of the SE of adjusted correlations was made. For the three-variable case, the bootstrap estimate and the two large-sample estimates provided in this study were compared. In general, the averages for the three estimates were close to one another and to the empirical standard deviation of the 250 adjusted correlations for each set of conditions when the sample size was 500. They were also relatively close to one another when the sample size was 100 and the selection ratio was .5 or .9. A specific example of this can be seen in Table 4 for samples from the population with the platykurtic distribution and the first correlation matrix. In this case for three variables, the full population correlation to be estimated was .25. When the size of the selected sample was 500, the averages of the 250 values of the three types of estimates for the SE were the same regardless of the selection ratio. These averages were generally the same as the empirical standard deviation of the 250 adjusted correlations, the exception being when the selection ratio was only .1. When the sample size was 100, the averages of the SE estimates were within .01 of one another. For instance, for selection ratio .1, the averages were .22, .23 and .23. These averages differed from the empirical standard deviation of .18 more than the averages for

selection ratios .5 and .9 differed from the empirical standard deviations of .10 and .09, respectively. In this example, when the sample size was 50 and the selection ratios were .5 and .9, the averages of the SE estimates and the empirical standard deviations were close to one another in value. When the sample size was 50 and the selection ratio was .1, larger differences between the average SE estimates and the empirical standard deviation occurred. In general, more differences between average SE estimates and the empirical standard deviation occurred when the sample size was 50, particularly for small selection ratios. For the two-variable case, the bootstrap estimate and the large-sample estimates were also close to one another and to the empirical standard deviation when the sample size was 500 and, in most cases, when the sample size was 100.

Second, information about the magnitudes of the SE's of adjusted correlations is available for the three-variable case. Examples of some of the relationships which hold can be seen in Table 4. Several of these relationships are expected on the basis of theory and intuition. For instance, given other conditions smaller SE's of the adjusted correlation were associated with larger sample sizes and with larger selection ratios (percentages of selected observations). For the two large-sample estimates this is to be expected since the formulas contain a factor of $N^{-\frac{1}{2}}$. It is of interest that a factor of $N^{-\frac{1}{2}}$ also appeared to account for differences among empirical standard deviations and the bootstrap SE's. An examination of the SE's for different selection ratios shows that differences between the SE's for selection ratios .1 and .5 were much larger than the differences between the SE's for selection ratios .5 and .9, particularly for the underlying beta distribution. Small values of the

Table 4

Average Estimates of Standard Errors
of the Adjusted Correlation Between X and $\hat{X}$.

Three-Variable Case
Underlying Platykurtic Distribution
Matrix $\underline{P}_1$
(.25 is being estimated)

Selection Ratio	N	Empirical SD	Bootstrap St. Error	1st Large-Sample St. Error	2nd Large-Sample St. Error
.1	50	.27	.31	.33	.33
	100	.18	.22	.23	.23
	500	.07	.08	.08	.08
.5	50	.15	.15	.15	.15
	100	.10	.10	.11	.11
	500	.05	.05	.05	.05
.9	50	.14	.13	.13	.14
	100	.09	.09	.09	.10
	500	.04	.04	.04	.04

ratio of the standard deviations of the selection variable before and after selection (S_z/s_z) occurred when the percentage of observations selected was large. Therefore, small SE's of the adjusted correlation correspond to small values of S_z/s_z. This was also seen in Table 5 which contains the empirical standard deviations for the three-variable case when the size of the selected sample was 500. Relationships between the population correlation matrix and the

magnitude of the SE, and between the underlying distribution and the magnitude of the SE can also be seen in Table 5. In general, given other conditions, smaller SE's of adjusted correlations correspond to the estimation of larger population correlations. The values of the SE's for $\underline{P}_3$, where .75 was being estimated were about half of the values of the SE's for $\underline{P}_2$, where .50 was being estimated. The values of the SE's for $\underline{P}_1$, where .25 was being estimated were close to the values of the SE's for $\underline{P}_2$. The large standard deviation of the 250 adjusted correlations from the platykurtic distribution with $\underline{P}_2$ when the selection ratio was .1 (.11) was due, upon examination, to the effects of stringent selection in the platykurtic case and the interplay between correlations in the population correlation matrix $\underline{P}_2$. Differences between the two underlying distributions will be further examined below.

A glance at Table 6 will indicate that, although the relationships between averages of the SE estimates and factors in the simulation design were similar for the two-variable case, the relationships were stronger for the two-variable case. As a matter of fact, when the correlation being estimated was the same, the magnitudes of the SE's of the adjusted correlation for the three-variable case were smaller than the magnitudes for the two-variable case. This appears to be primarily due to the additional information available at the extremes of the bivariate distribution of the two indirectly-selected variables for the three-variable case. In summary, given other conditions, smaller SE's were associated with smaller values of S_z/s_z, with larger sample sizes, with larger selection ratios, and, ordinarily, with larger underlying correlations.

Table 5

Standard Deviation
of 250 Adjusted Correlations between X and Y.

Three-Variable Case
N=500

Distribution	Matrix	Population Correlation	Selection Ratio .1	.5	.9
Normal	$\underline{P}_1$	.25	.05	.05	.04
	$\underline{P}_2$	.50	.05	.04	.04
	$\underline{P}_3$	.75	.03	.02	.02
Platykurtic	$\underline{P}_1$	.25	.07	.05	.04
	$\underline{P}_2$	.50	.11	.05	.03
	$\underline{P}_3$	.75	.05	.02	.02

Table 6

Standard Deviation
of 250 Adjusted Correlations between X and Z.

Two-Variable Case
N=500

Distribution	Matrix	Population Correlation	Selection Ratio .1	.5	.9
Normal	$\underline{P}_1$	.20	.10	.07	.05
	$\underline{P}_2$	.50	.07	.05	.04
	$\underline{P}_3$	.80	.03	.02	.02
Platykurtic	$\underline{P}_1$	.20	.19	.08	.05
	$\underline{P}_2$	.50	.14	.05	.04
	$\underline{P}_3$	.80	.05	.02	.01

Third, information about the SE of adjusted correlations for data sets selected from a platykurtic population was obtained. In general, the relationships between design characteristics and the magnitudes of the SE's for data sets associated with the beta distribution were similar to those associated with the normal distribution. Specifically, expected relationships between the magnitude of the SE and sample size, population correlations and selection ratios were observed. Although relationships could be identified for each underlying distribution, the role of S_z/s_z was more clear when SE's for the two types of distributions could be compared. As stated previously, the magnitude of S_z/s_z is small for large selection ratios. This can be seen in Table 7 in terms of the population value Σ_z/σ_z for the two types of distributions. The difference between magnitudes of Σ_z/σ_z for the two different distributions of the selection variable can also be seen in Table 7. When selection is not stringent, Σ_z/σ_z for the normal distribution and for the beta distribution are close in magnitude. However, when the selection ratio is .1, the value of Σ_z/σ_z for the beta distribution is almost twice as large as the value for the normal distribution. As expected, the values in both Table 5 and Table 6 show that the corresponding empirical standard deviations of the adjusted correlation for the normal and platykurtic distributions are similar in value for selection ratios .5 and .9. However, for the selection ratio .1, the standard deviations of the adjusted correlation for the platykurtic distribution are noticeably larger in value than the standard deviations of the adjusted correlation for the normal distribution. To the extent that test scores are curtailed and platykurtic, as are the observations from the beta (2,2) distribution, concern over SE's of adjusted correlations is heightened. However, contrary to the statement by

Lord and Novick (1968, p. 147) that a value of Σ_z/σ_z larger than 1.4 is severe enough to cause concern in using the Pearson-Lawley adjustments, the standard deviations in Table 5 indicate that when the sample size is large the standard deviations are reasonably small in relation to the size of the correlation being estimated, even when selection is more stringent than 70 percent selection. If concern about the use of the Pearson-Lawley adjustments when Σ_z/σ_z is larger than 1.4 is well-founded, it must be based, as Lord and Novick alluded, on the size of the bias of the adjustments when the assumptions are not strictly met.

Table 7

Ratio of the Standard Deviations of the Selection Variable in the Full Population and in the Population Fitting the Selection Criterion (Σ_z/σ_z)

Distribution of the Selection Variable	Selection Ratio .1	.5	.9
Normal (0,1)	2.43	1.66	1.19
Standardized Beta (2,2)	4.73	1.84	1.14

Conclusions

Selection occurs in many real-life settings in education and in measurement. In particular, when validity evidence is provided for tests used to select applicants or for tests administered to examinees selected for some other reason, correlations are often used. On the basis of this study, two suggestions can be made to practitioners using the Pearson-Lawley adjustments in these settings. The first is a recommendation for the use of the estimates for

the standard error of the adjusted correlations. The second is a warning concerning the use of an adjusted correlation without reporting an estimate of its SE, and is based upon the magnitude of the standard error in a variety of settings, including those for the platykurtic distribution and for the three-variable case. Since these suggestions are made in the context of an exact fit of the data to the Pearson-Lawley assumptions of linearity and homogeneity of variance, they indicate "best case" results for real data which may not fit the assumptions exactly.

The dominant finding from comparisons of the alternative estimates of standard errors was that for settings in which one might have some degree of confidence in the Pearson-Lawley adjustments, the SE estimates provided very similar assessments of the degree of sampling error. Given these results, the use of the simpler of the two large-sample estimates is suggested for both the two- and three-variable cases when the sample size is large. Although the assumptions for this estimate may not be strictly true (the multivariate distribution underlying unadjusted correlations is not usually mesokurtic), this approximation appeared to offer a reasonable estimate of the SE of an adjusted correlations without resorting to the computer-intensive approach of the bootstrap estimate or to the complexity of the large-sample estimate with less stringent assumptions.

An examination of the magnitudes of the estimates for the SE of the three-variable adjusted correlations indicated that even when the sample size was 100, adjusted correlations were relatively variable when the selection ratio was .1. The empirical standard deviation of the 250 adjusted correlations for the normal data sets with a population correlation of .50, for instance, was .13. For the

set of conditions described above, the empirical standard deviation was .26 when the underlying distribution was platykurtic. The SE of an adjusted correlation was always large relative to the correlation itself when the selection ratio was .1, but particularly when the underlying distribution was platykurtic, when the population correlations were small or when the sample size was less than 500. For the two-variable case, an examination of the relative magnitudes of the standard deviations showed that caution in the use of the adjusted correlation as a simple point-estimate for small selection ratios was also in order.

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Appendix
Average Estimates of Standard Errors of the Adjusted Correlation Between X and Y (Three-Variable Case)

DISTRIBUTION	MATRIX	N	SELECTION RATIO .1 SD	.1 BTSE	.1 LSE1	.1 LSE2*	.5 SD	.5 BTSE	.5 LSE1	.5 LSE2	.9 SD	.9 BTSE	.9 LSE1	.9 LSE2
Normal	P1	50	.18	.19	.19	.19	.15	.15	.15	.15	.14	.13	.13	.13
		100	.11	.12	.12	.12	.10	.10	.10	.10	.10	.09	.10	.10
		500	.05	.05	.05	.05	.05	.05	.05	.05	.04	.04	.04	.04
	P2	50	.18	.19	.19	.19	.14	.14	.14	.14	.12	.11	.11	.11
		100	.13	.13	.13	.13	.10	.10	.10	.10	.08	.08	.08	.08
		500	.05	.06	.06	.06	.04	.04	.05	.04	.04	.03	.04	.04
	P3	50	.10	.11	.10	.09	.08	.07	.08	.07	.07	.06	.07	.07
		100	.07	.07	.07	.06	.05	.05	.05	.05	.05	.04	.05	.05
		500	.03	.03	.03	.03	.02	.02	.02	.02	.02	.02	.02	.02
Beta	P1	50	.27	.31	.33	.33	.15	.15	.15	.15	.14	.13	.13	.14
		100	.18	.22	.23	.23	.10	.10	.11	.11	.09	.09	.09	.10
		500	.07	.08	.08	.08	.05	.05	.05	.05	.04	.04	.04	.04
	P2	50	.36	.30	.32	.32	.15	.15	.15	.15	.11	.11	.11	.12
		100	.26	.23	.24	.24	.11	.11	.11	.11	.08	.07	.08	.08
		500	.11	.11	.11	.11	.05	.05	.05	.05	.03	.03	.04	.04
	P3	50	.25	.23	.20	.20	.08	.07	.08	.09	.06	.05	.07	.07
		100	.12	.15	.13	.13	.05	.05	.06	.06	.04	.04	.05	.05
		500	.05	.05	.05	.05	.02	.02	.02	.03	.02	.02	.02	.02

* SD is the standard deviation of the 250 replications in each cell, BTSE is the bootstrap estimate, LSE1 is the first large-sample estimate and LSE2 is the second large-sample estimate.

Average Estimates of Standard Errors of the Adjusted Correlation Between X and Z (Two-Variable Case)

DISTRIBUTION	MATRIX	N	SELECTION RATIO .1 SD	.1 BTSE	.1 LSE1	.1 LSE2*	.5 SD	.5 BTSE	.5 LSE1	.5 LSE2	.9 SD	.9 BTSE	.9 LSE1	.9 LSE2
Normal	P1	50	.32	.28	.29	.29	.22	.21	.21	.21	.16	.15	.16	.16
		100	.21	.22	.22	.22	.14	.15	.15	.15	.11	.11	.11	.11
		500	.10	.10	.10	.10	.07	.07	.07	.07	.05	.05	.05	.05
	P2	50	.23	.23	.23	.23	.18	.16	.16	.16	.13	.12	.12	.12
		100	.16	.16	.16	.16	.11	.12	.12	.12	.08	.08	.08	.08
		500	.07	.07	.07	.07	.05	.05	.05	.05	.04	.04	.04	.04
	P3	50	.10	.11	.10	.09	.07	.07	.07	.07	.06	.05	.06	.06
		100	.07	.07	.07	.06	.05	.04	.05	.04	.04	.03	.04	.04
		500	.03	.03	.03	.03	.02	.02	.02	.02	.02	.02	.02	.02
Beta	P1	50	.50	.41	.45	.45	.23	.22	.23	.23	.16	.15	.15	.15
		100	.40	.33	.36	.36	.18	.16	.17	.17	.11	.11	.12	.12
		500	.19	.18	.19	.19	.08	.08	.08	.08	.05	.05	.05	.05
	P2	50	.46	.37	.40	.40	.17	.17	.17	.18	.11	.11	.12	.12
		100	.34	.30	.31	.31	.13	.12	.13	.13	.08	.08	.08	.09
		500	.14	.14	.14	.14	.05	.05	.06	.06	.04	.04	.04	.04
	P3	50	.26	.24	.20	.20	.07	.07	.07	.08	.05	.04	.05	.07
		100	.13	.16	.13	.13	.04	.05	.05	.05	.03	.03	.04	.05
		500	.05	.05	.05	.05	.02	.02	.02	.02	.01	.01	.02	.02

* SD is the standard deviation of the 250 replications in each cell, BTSE is the bootstrap estimate. LSE1 is the first large-sample estimate and LSE2 is the second large-sample estimate.